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|  Written Maths Calculation Policy  for Lickey Hills Primary and Nursery School. September 2014. |

 

**Progression towards a standard written method of calculation.**

 **INTRODUCTION**

This calculation policy has been written in line with the programmes of study taken from the revised **National Curriculum for Mathematics (2014).** It provides guidance on appropriate calculation methods and progression. The content is set out in yearly blocks under the following headings: addition, subtraction, multiplication and division.

Children will use mental methods as their first port of call when appropriate, but for calculations that they cannot do in their heads, they will need to use an efficient written method accurately and with confidence.

 **AIMS OF THE POLICY**

* To ensure consistency and progression in our approach to calculation
* To ensure that children develop an efficient, reliable, formal written method of calculation for all operations
* To ensure that children can use these methods accurately with confidence and understanding.

**HOW TO USE THIS POLICY**

* Use the policy as the basis of your planning but ensure you use previous or following years’ guidance to allow for personalised learning
* Always use Assessment for Learning to identify suitable next steps in calculation for groups of children
* If, at any time, children are making significant errors, return to the previous stage in calculation
* Always use suitable resources, models and images to support children’s understanding of calculation and place value
* Encourage the children to make sensible choices about the methods they use when solving problems
* Develop key vocabulary used in both mental and written calculations.

 **STAGES IN ADDITION**

**Addition-Early Stages (EYFS)**

Children will engage in a wide variety of songs and rhymes, games and activities. They will begin to relate addition to **combining two groups of objects,** first by **counting all** and then by **counting on** from the largest number.

They will find one more than a given number.

In practical activities and through discussion they will begin to use the vocabulary involved in addition.

   

“You have 5 apples and I have 3 apples. How many apples altogether?”

**Addition-Year One**

* **Given a number, identify one more**
* **Read, write and interpret mathematical statements involving addition (+) and the equals (=) sign**
* **Add one-digit and two digit numbers within 20, including zero**
* **Solve missing number problems eg 10 + ? = 16.**

**NB** Ensure that children are confident with the methods outlined in the previous year’s guidance before moving on.

Children will continue to practise counting on from any number eg “ Put five in your head and count on four.”

Initially, use a **number track** to count on for addition, counting on from the largest number.

|  |
| --- |
|  **1 2 3 4 5 6 7 8 9 10** |

**5 + 4 = 9**

“Put your finger on number five. Count on (count forwards) four.”

Then progress to a **marked number line:**

**6 + 6 = 12**



“Put your finger on six and count on six.”

Ensure children are confident with using a marked number line before moving on to an empty number line (see Year Two guidance).

Continue to practise counting on from the largest number for addition with totals within 20.

**Addition-Year Two.**

* **Add numbers using concrete objects, pictorial representations, and mentally, including:**
* **A two digit number and ones**
* **A two digit number and tens**
* **Two two-digit numbers**
* **Three one digit numbers.**

**NB** Ensure that children are confident with the methods outlined in the previous year’s guidance before moving on.

Counting on in ones using an **empty number line,** within 100.



**28 + 6 = 34**

….and in tens

**18 + 30 = 48**

Use in conjunction witha **100 square** to show jumps of tens.

**48 + 36 = 84**

Use a marked number line in conjunction with a **100 square** to show jumps of tens and ones.

“Put the bigger number first(48), then partition the smaller number (36= 30 +6) and count on: 48 + 10 + 10 + 10 + 1 +1 +1 +1 +1 +1 = 84.” (**48+30+6=84**)

If children are confident, use more efficient jumps…

Eg **48 + 30 + 2 + 4 = 84**

Use in conjunction with a **hundred square** to show jumps of tens and ones/units.

Also use the **partitioning method** to add two two-digit numbers:

 **43 + 25 = 68**

**40 + 20 = 60 3 + 5 = 8 60 + 8 = 68**

“Partition the numbers into tens and ones/units.

Add the tens together and then add the units together.

Recombine to give the answer.”

Then move onto calculations that **bridge** the tens:

 **48 + 36 = 84**

**30 + 40 = 70 8 + 6 = 14 70 + 14 = 84**

Further develop addition with numbers that bridge 100, using a 200 grid to support.

If, at any time, children are making significant errors, return to the previous stage in calculation.

**Addition-Year Three.**

* **Add numbers with up to three digits, using formal written method of columnar addition.**

**NB** Ensure that children are confident with the methods outlined in the previous year’s **guidance** before moving on.

Further develop the use of the **empty number line** with calculations that **bridge 100.**

**78 + 46 = 124**

**70 + 40 = 110 8 + 6 = 14 110 + 14 = 124**

Use a **200 grid** to support counting on in tens and bridging 100…..

……and with addition of a three-digit number and a two-digit number:

**165 + 56 = 221**

**165 + 50 = 215 215 + 6 = 221**

Further develop the **partitioning method** with calculations that **bridge 100:**

**85 + 37 = 122**

**80 + 30 = 110 5 + 7 = 12 110 + 12 = 122**

The partitioning method can also be used with three-digit numbers.

Introduce the **expanded written method** with the calculation presented both horizontally and vertically (in columns).

Initially, use calculations where it has not been necessary to bridge across the tens or hundreds:

**63 + 32 = 95**

 **60 + 3** “Partition the numbers into tens and units.

 **+ 30 + 2** Add the tens together and then add the

 **90 + 5 = 95** units. Recombine to give the answer.

Then….

 **63** Add the least significant digits (units)

 **+ 32**  together first and then the tens in

 **5 ( 3+ 2)** preparation for the formal written method.

 **+ 90 ( 60 + 30)**

 **95**

This will lead into the **formal written method….**

 **63** Use the language of place value to ensure

 **+ 32**  understanding:

 **95** “Three add two equals five. Write five in

 The units column. 60 add 30 equals 90.

 Write 9 (90) in the tens column.”

**NB** Informal/mental methods would be more appropriate for numbers of this size, but use two-digit numbers when introducing the columnar method.

Then introduce calculations where it is necessary to bridge, returning to an **expanded method** initially:

**68 + 24 = 92**

 **60 + 8** “Partition the numbers into tens and units.

 **+ 20 + 4**  Add the tens together and then the units

 **80 + 12 = 92** together. Recombine to give the answer.”

**Then……**

 **68** Add the least significant digits (units)

 **+ 24** together first and then the tens in

  **12 (8 + 4)** preparation for the normal written

 **80 (60 + 20)** method.

 **92**

**If the children are ready,** introduce the **formal written method,** where it is necessary to “carry” ten from the units column to the tens column.

 **68** Use the language of place value to ensure

 **+ 24** understanding. “ Eight add four equals 12.

1. Write two in the units column and “carry” one (10)
2. across into the tens column. 60 add 20 and the ten that

we have “carried” equals 90. Write 9 (90) in the tens

column. The answer is 92.

The digit that has been “carried” should be recorded under the line in the correct column.

**When the children are confident**, extend with examples where it is necessary to bridge across the tens and hundreds:

 **76 + 47 = 123**

 **70 + 6** “Partition the numbers into tens and ones/units.

 **+ 40 + 7** Add the tens together and then the ones/units

 **110 + 13 = 123** together. Recombine to give the answer.”

Then…..

 **76** Add the least significant digits (units) together

 **+ 47** first and then the tens in preparation for the

 **13 ( 7 + 6)** formal written method.

 **+ 110 ( 70 + 40)**

 **123**

**If the children are ready,** introduce the **formal written method,** where it is necessary to “carry” across the columns and bridge 100:

 **76 + 47 = 123**

 **76** Use the language of place value to ensure understanding:

 **+ 47** “Six add seven equals 13. Write three in the units column

 **123** and “carry” one (10) across into the tens column. 70 add 40

 1 1 and the ten that we “carried” equals 120. Write 2 (20) in

 the tens column and “carry” one (100) across into the

 hundreds column (100).”

The digits that have been “carried” should be recorded under the line in the correct column.

**If the children are confident,** further develop with the addition of a three-digit number and a two digit number:

 **178 + 43 = 221**

 **178**

 **+ 43**

 **221**

1. 1

**NB** If, at any time, children are making significant errors, return to the previous stage in calculation.

**Addition-Year Four**

* **Add numbers with up to 4 digits using the formal written method of columnar addition where appropriate.**

**NB** Ensure that children are confident with the methods outlined in the previous year’s guidance before moving on.

Continue to teach the use of **empty number lines** with three and four digits, as appropriate.

Further develop the formal written method of addition, with three –digit numbers. Revisit the expanded method first, if necessary.

 **176 + 147 = 323**

 **176**

 **+ 147**

 **13 ( 6 + 7)**

 **110 ( 70 + 40)**

 **200 ( 100 + 100)**

 **323**

This will lead into the **formal written method….**

 **176 + 147 = 323** Use the language of place value to ensure understanding:

 **“** Six add seven equals 13. Write the three in the units

 **176** column and “carry” one (10) across into the tens column.

 **+ 147**  70 add 40 and the ten that was “carried” equals 120.

 **323** Write 2 in the tens column (20) and “carry” one across

 1 1 into the hundreds column (100). 100 add 100 and the 100

 that has been “carried” equals 300. Write 3 in the

 hundreds column (300).

The digits that have been “carried” should be recorded under the line in the correct column.

**If the children are confident,** introduce the addition of a four- digit and a three-digit number:

 **1845 + 526 = 2371**

 **1845**

 **+ 526**

 **2371**

1. 1

Continue to develop with addition of two four-digit numbers and with decimals (in the context of money or measures).

**NB** If, at any time, children are making significant errors, return to the previous stage in calculation.

**Addition-Year Five**

* **Add whole numbers with more than 4 digits, including using formal written method ( columnar addition)**

**NB** Ensure that children are confident with the methods outlined in the previous year’s guidance before moving on.

Continue to teach the use of **empty number lines** with larger numbers (and decimals), as appropriate.

Continue to develop the **formal written method for addition** with larger numbers (and decimal numbers) and with the addition of three or more numbers:

 **21848 + 1523 = 23371**

 **21848**

 **+ 1523**

 **23371**

1. 1

Continue to use the language of place value to ensure understanding. Ensure that the digits that have been “carried” are recorded under the line in the correct column.

Use the **formal written method** for the addition of decimal numbers:

 **£154.75 + £233.82 = £388.57**

 **154.75** Continue to use the language of place value

 **+ 233.82** to ensure understanding.

 **388.57**

Ensure that the decimal points line up.

Continue to practise and apply the formal written method throughout Year Five.

**NB** If, at any time, children are making significant errors, return to the previous stage in calculation.

**Addition-Year Six**

No objectives have been included in the programmes of study explicitly related to written methods for addition in Y6. However, there is an expectation that children will continue to practise and use the **formal written method for larger numbers and decimals** and use these methods when solving problems, when appropriate (see previous year’s guidance for methods).

Our aim is that, by the end of Year Six, children **use mental methods (with jottings)** when appropriate, but for calculations that they cannot do in their heads, they use an efficient **formal written method** accurately and with confidence.

 **STAGES IN SUBTRACTION.**

**Subtraction-Early Stages (EYFS)**

Children will engage in a variety of counting songs and rhymes and practical activities.

In practical activities and through discussion, they will begin to use the vocabulary associated with subtraction.

They will find one less than a given number.

They will begin to relate subtraction to “taking away” **using objects** to count ” how many are left” after some have been taken away.

 **6 – 2 = 4**    

“Take two apples away. How many are left?”

Children will begin to count back from a given number.

**Subtraction-Year One**

* **Given a number, identify one less**
* **Read, write and interpret mathematical statements involving subtraction (-) and the equals sign (=)**
* **Subtract one-digit and two-digit numbers within 20, including zero**
* **Solve missing number problems eg 20 - ? = 15**

**NB** Ensure that children are confident with the methods outlined in the previous year’s guidance before moving on.

Children will continue to practise counting back from a given number.

Initially use a **number track** to **count back** for subtraction.

|  |
| --- |
|  **1 2 3 4 5 6 7 8 9 10** |

**9 – 5 = 4** “Put your finger on number nine. Count back five.”

Then progress to a **marked number line:**

**12 – 6 = 6**

 

 “Put your finger on number twelve and count back six.”

**14 – 5 = 9**

**Using a marked number line,** “Put your finger on fourteen and count back five.”

**NB** Ensure children are confident with using a **marked number line** before moving on to an **empty number line** (see Year Two guidance).

Continue to practise counting back for subtraction with numbers within 20.

**Counting on to find a small difference:**

Introduce complementary addition to find differences (only use for **small** differences). The use of models is extremely important here to understand the idea of “difference”.

**Count up** from the smaller number to the larger number to find the difference using resources eg cubes, beads, number tracks/lines:

 **11 – 9 = 2**





The **difference between** nine and eleven is two.

**NB** If, at any time, children are making significant errors, return to previous stage in calculation.

**Subtraction- Year Two**

* **Subtract numbers using concrete objects, pictorial representations, and mentally, including:**
* **A two-digit number and ones**
* **A two-digit number and tens**
* **Two two-digit numbers**

**NB** Ensure that children are confident with the methods outlined in the previous year’s guidance before moving on.

Counting back using an **empty number line** within 100, in ones…….

**34 – 6 = 28**



and in tens:

**58 – 30 = 28**

Use in conjunction with a **100 square** to show jumps of ten.

**Subtraction, using partition, on an empty number line:**

**76 – 45 = 31**

 **-10 -10 -10 -10**

**-1 -1 -1 -1 -1**

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31 32 33 34 35 36 46 56 66 76

Use in conjunction with a **100 square** to show jumps of tens and ones.

If the children are confident, use more efficient jumps:

**76 – 45 = 31**

 **-5 -40**

 31 36 76

**76 – 40 – 5 = 31**

Use in conjunction with a **100 square** to show jumps of tens and ones.

**Counting on to find a small difference:**

Introduce complementary addition to find differences ( only use for **small** differences). The use of models is extremely important here to understand the idea of “difference”( see Year One guidance).

**Count up** from the smaller number to the larger number to **find the difference.**

**12 – 8 = 4**

 **+1 +1 +1 +1**

**8 9 10 11 12**

“The difference between 8 and 12 is 4.”

Use number lines and 100 squares to develop the idea with larger numbers:

**32 – 28 = 4**

Use jumps of one to show that the difference between 28 and 32 is 4.

**If children are confident**, further develop this method:

**76 – 58 = 18**

 **+2 +10 +6**

**58 60 70 76**

“The difference between 58 and 76 is 18.”

Further develop subtraction with numbers that bridge 100, using a **200 grid** to support.

**NB** If, at any time, children are making significant errors, return to the previous stage of calculation.

**Subtraction-Year Three**

* **Subtract numbers with up to three digits, using formal written methods of columnar subtraction.**

**NB** Ensure that children are confident with the methods outlined in the previous year’s guidance before moving on.

Further develop the use of the **empty number line** with calculations that **bridge 100:**

**126 – 45 = 81**

 **-5 -10 -10 -10 -10**

**81 86 96 106 116 126**

Use a **200 grid** to support counting back in tens bridging 100

Then use more efficient jumps:

 -5 -40

 **81 86 126**

Extend with larger numbers counting back:

**216 – 27 = 189**

Use a number line as above to show:

**216 – 20 = 196 196 – 6 = 190 190 – 1 = 189**

**………..**and by **counting on to find the difference** (small difference):

**231 – 198 = 33**

 +2 +30 +1

198 200 230 231

“The difference between 198 and 231 is 33.”

Introduce the **expanded written method** with the calculation presented both horizontally and vertically (in columns).

**78 – 23 = 55** “Partition the numbers into tens and ones/units.

Subtract the ones and then the tens.

 **70 + 8** Recombine to give the answer.”

* **20 + 3 NB** In this example, decomposition (exchange)

 **50 + 5 = 55** is not required.

You might want to replace the **+ sign** with the word **“and”** to avoid confusion.

This will lead into the **formal written method:**

 **78** Use the language of place value to ensure

* **23**  understanding. “Eight subtract three, seventy subtract twenty.”

 **55**

**NB** A number line would be an appropriatemethod for this calculation but use two digit numbers to illustrate the formal written method initially.

Introduce the **expanded written method** where **exchange/decomposition** is required:

**73 – 27 = 46** Use the language of place value to ensure

 6 13 understanding. “We can’t subtract 7 from 3, so

 **~~7~~ 3** we need to exchange a ten for ten ones to give

* **2 7**  us 60 + 13.”

 **4 6** Use **base ten materials** to support understanding.

**If children are confident,** extend the use of the **formal written method with numbers over 100,** returning to the expanded method first, if necessary.

**235 – 127 = 108** Use the language of place value to ensure

understanding. In this example it has only

 2 15 been necessary to exchange from the tens

  **2~~3~~5** column.

 **- 127**  Use **base ten materials** to support understanding.

 **108**

**NB** If, at any time, children are making significant errors, return to the previous stage in calculation.

**Subtraction-Year Four**

* **Subtract numbers with up to four digits using the formal written method of columnar subtraction where appropriate.**

**NB** Ensure that children are confident with the methods outlined in the previous year’s guidance before moving on.

Continue to teach the use **of empty number lines** with three and four digit numbers as appropriate.

Continue to develop the **formal written method of subtraction** by revisiting the **expanded method** first, if necessary. Continue to use **base ten materials** to support understanding.

**258 – 73 = 185**

 **200 + 50 + 8 100 + 150 + 8**

* **70 + 3 becomes - 70 + 3**

 **100 + 80 + 5 = 185**

You might want to replace the **+ sign** with the word **“and”** to avoid confusion. Children will need to practise partitioning in a variety of ways. This leads to the **formal written method,** involving decomposition.

 1 15

 **~~2~~58** Use the language of place value to ensure understanding.

* **73** In this example, it has been necessary to exchange from

 **185** the hundreds column.

Further develop by subtracting a three-digit number from a three-digit number:

 **637 – 252 = 385**

 **600 + 30 + 7 500 + 130 + 7**

 **- 200 + 50 + 2 200 + 50 + 2**

 **300 + 80 + 5 = 385**

Ensure that the children are confident partitioning numbers in this way.

This leads to a **formal written method:**

 5 13

 **~~6~~37** Use the language of place value to ensure

 **- 252**  understanding and use base-ten materials

 **385**  if necessary.

**When the children are confident**, develop with **four-digit numbers** and decimal numbers (in the context of money and measures).

 **3625 – 1219 = 2406**

 1 15

 **36~~2~~5**

 **- 1219**

 **2406**

**NB** If, at any time, children are making significant errors, return to the previous stage in calculation.

**Subtraction- Year 5**

* **Subtract whole numbers with more than four digits, including formal written method (columnar subtraction)**

**NB** Ensure that children are confident with the methods outlined in the previous year’s guidance before moving on.

Continue to teach the use of **empty number lines** with larger numbers and decimals as appropriate.

Continue to develop the **formal written method for subtraction** with three and four digit numbers (see Y4 guidance), returning to an expanded method and using base ten materials, if necessary.

 **503 – 278 = 225**

 **500 + 0 + 3 400 + 90 + 13**

**- 200 +70 + 8** **200 + 70 + 8**

 **200 + 20 + 5**

In this example, 503 has to be partitioned into 400+90+13 in order to carry out the subtraction calculation.

This leads to the **formal written method.** There is potential for error in this example.

 4 9 13

 **~~50~~3** There are no tens in the first number (503) so we have

 **- 278**  to exchange a hundred for 10 tens before we can exchange a ten

 **225** for ten ones/units.

**NB** It would be appropriate to discuss the use of mental calculation methods with an example like this one i.e. would an empty number line be a more efficient method for these numbers?

**When the children are confident** extend with larger numbers (and decimal numbers). Return to an expanded method if necessary.

 **12731 – 1367 = 11364**

 6 12 11

 **12~~73~~1** In this example it has been necessary to exchange

 **- 1367** from the tens and hundreds columns.

 **11364**

**NB** If children are making significant errors, provide calculations where only one exchange is needed.

Introduce subtraction of decimals, initially in the context of money and measures.

**£166.25 - £83.72 = £82.53**

 16 5 12

 **166.25** Ensure the decimal points line up.

**- 83.72**

 **82.53**

Continue to practise and apply the formal written method with large numbers and decimals throughout Y5.

**NB** If, at any time, children are making significant errors, return to the previous stage in calculation.

**Subtraction-Year Six**

No objectives have been included in the programmes of study explicitly related to written methods for subtraction in Y6. However, there is an expectation that children will continue to practise and use **the formal written method for large numbers and decimals** and use these methods when solving problems as appropriate (see previous year’s guidance for methods).

Our aim is that, by the end of Y6, children **use mental methods (with jottings)** when appropriate, but for calculations that they cannot do in their heads, they use an efficient **formal written method** accurately and with confidence.

 **STAGES IN MULTIPLICATION.**

**Multiplication- Early Years (EYFS)**

Children will engage in a wide variety of songs and rhymes, games and activities. In practical activities and through discussion, they will begin to solve problems involving doubling.



“Three apples for you and three apples for me. How many apples altogether?”

**Multiplication-Year One**

* **Solve one step problems involving multiplication by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher**
* **Count in multiples of twos, fives and tens** (to the tenth multiple).

Children will count repeated groups of the same size in practical contexts.They will use the vocabulary associated with multiplication in practical contexts. They will solve **practical problems** that involve combining groups of 2, 5 or 10 e.g socks, fingers and cubes.



“Six pairs of socks. How many socks altogether?

2, 4, 6, 8, 10, 12”

  

“Three sets of ten dogs. How many dogs altogether?”

Use **arrays** to support early multiplication.

 

   

“Five groups of two faces. How many faces altogether? 2, 4, 6, 8, 10

Two groups of five faces. How many faces altogether? 5, 10”

 

“2 groups of 5”

“How many altogether?”

“5 + 5 = 10”

“Double five is ten.”

Continue to solve problems **in practical contexts** and develop the language of early multiplication, with appropriate resources, throughout Y1.

**Multiplication-Year Two**

* **Recall and use multiplication facts for the 2, 5 and 10 multiplication tables**
* **Calculate mathematical statements for multiplication within the multiplication tables and write them using the multiplication (x) and equals (=) signs**
* **Solve problems involving multiplication, using materials, arrays, repeated addition, mental methods and multiplication facts, including problems in context**
* **Show that multiplication of two numbers can be done in any order (commutative)**

**NB** Ensure that children are confident with the methods outlined in the previous year’s guidance before moving on.

Children will use a range of vocabulary to describe multiplication and use practical resources, pictures, diagrams and the x sign to record.

**Combining Groups (repeated addition):**

  

“3 groups of 10 butterflies”

“How many butterflies altogether?”

“10 + 10 + 10 = 30”

“3 groups of 10” “3 times 10”

“3 x 10 = 30” “10 x 3 = 30”



  

“5 groups of 3” “5 lots of 3” “ 3 + 3 + 3 + 3 + 3 = 15”

“5 times 3” “3 multiplied by 5” “ 5 x 3 = 15” “3 x 5 = 15”

**Use arrays to support multiplication**

**6 x 5 =30**

 “5 + 5 + 5 + 5 + 5 + 5 = 30”

 “6 rows of 5”

 “6 groups of 5”

 “ 5 groups of 6”

 “ 5 x 6 = 30”

 **“** 6 x 5 = 30”

**Use an empty number line:**

**6 x 5 = 30**

 **1x5 2x5 3x5 4x5 5x5 6x5**

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**0 5 10 15 20 25 30**

Make the link to repeated addition.

**NB** If, at any time, children are making significant errors, return to the previous stage in calculation.

**Multiplication- Year Three**

* **Recall and use multiplication facts for the 3, 4 and 8 multiplicaton tables** (continue to practise the 2, 5 and 10 multiplication tables)
* **Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to a formal written method.**

**NB** Ensure that the children are confident with the methods outlined in the previous year’s guidance before moving on.

Continue to use **number lines** and **arrays** to support multiplication, as appropriate (see Y2 guidance).

**4 x 3 =12**

1x3 2x3 3x3 4x3

**0 3 6 9 12**

**Partitioning method for multiplication of a teen number by a one-digit number:**

**13 x 5 = 65**

**10 x 5 = 50**

**3 x 5 = 15**

**13 x 5 = 50 + 15 = 65**

Demonstrate the partitioning method using a **number line:**

**13 x 5 = 65**

10 x 5 = 50 3 x 5 = 15

**0 50 65**

**Grid method** (teen number multiplied by a one-digit number):

**13 x 8 = 104**

|  |  |  |
| --- | --- | --- |
|  **X** |  **10** |  **3** |
|  **8** |  **80**  |  **24** |

**80 + 24 = 104**

**“Partition 13 into 10 + 3 then multiply each number by 8. Add the partial products ( 80 and 24) together.”**

This will lead into **expanded short multiplication:**

**13 x 8 =104**

**10 + 3**

**X 8**

 **24 (3 x 8)** Include an addition symbol when adding

**+ 80 (10 x 8)**  partial products.

 **104**

Refine the recording process in **preparation for formal short multiplication:**

**13 x 8 = 104**

 **13** Use the language of place value to ensure

**X 8** understanding.

 **24 (3 x 8)** Include an addition symbol when adding

 **+ 80 (10 x 8)** partial products.

 **104**

Model the same calculation using a number line, if necessary, to ensure understanding.

**Formal short multiplication:**

 **13** Ensure that the digit “carried over” is written under the

**X 8** line in the correct column.

 **104** Use the language of place value to ensure understanding.

Continue to develop the formal written method of multiplication throughout Y3 using teen-numbers multiplied by a one-digit number.

**If children are confident,** progress to multiplying other two-digit numbers by a one-digit number (see Y4 guidance).

**NB** If, at any time, children are making significant errors, return to the previous stage in calculation.

**Multiplication-Year Four**

* **Recall multiplication facts for multiplication tables up to 12 x 12**
* **Multiply two-digit and three-digit numbers by a one-digit number using formal written layout**

**NB** Ensure that children are confident with the methods outlined in the previous year’s guidance before moving on.

Continue to use **empty number lines** **,** as appropriate **(**see Y3 guidance).

Further develop the **grid method** for two-digit numbers multiplied by a one-digit number.

**36 x 4 = 144**

|  |  |  |
| --- | --- | --- |
|  **X** |  **30** |  **6** |
|  **4**  |  **120** |  **24** |

 **120 + 24 = 144 (add the partial products)**

**Expanded short multiplication** (two-digit number by a one-digit number):

**36 x 4 = 144**

 **30 + 6**

**X 4**

 **24** (4 x 6) Include an addition symbol when adding

 **+ 120** (4 x 30) partial products.

 144

Refine the recording in preparation for formal short multiplication:

**36 x 4 = 144**

 **36**

**X 4**

**+ 24** (4 x 6)

 **120** (4 x 30)

 **144**

This leads to **short multiplication (formal method)** of a two-digit number multiplied by a one-digit number:

 **36** Use the language of place value to ensure understanding.

**X 4** Ensure that the digit “carried over” is written under the line in the

 **144** correct column.

Continue to practise the formal method of short multiplication of a two-digit number by a one-digit number throughout Y4.

**If the children are confident,** continue to develop short multiplication with three-digit numbers multiplied by a one-digit number.

If necessary, return to **the grid method and/or expanded method** first:

**127 x 6 = 762**

|  |  |  |  |
| --- | --- | --- | --- |
|  **X** |  **100** |  **20** |  **7** |
|  **6** |  **600** |  **120** |  **42** |

 **600 + 120 + 42 = 762** (add the partial products).

This leads to **expanded short multiplication:**

 **127**

**X 6**

 **42** (6 x 7)

 **120** (6 x 20)

 **600** (6 x 100)

 **762**

This will lead into **short multiplication (formal method):**

 **127** Use the language of place value to ensure understanding.

**X 6**  Ensure that the digits “carried over” are written under the

 **762** line in the correct column.

 1 4

**NB** If, at any time, children are making significant errors, return to the previous stage in calculation.

**Multiplication-Year Five**

* **Multiply numbers up to four digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers**

**NB** Ensure that children are confident with the methods outlined in the previous year’s guidance before moving on.

Build on the work covered in Y4 with the **formal written method of short multiplication** (two-digit number multiplied by a one-digit number).

**When children are confident introduce multiplication by a two-digit number.** If necessary, return to the grid method and/or expanded method first.

**Grid method** (two-digit by a teen-number):

**23 x 13 = (20+3) x (10+ 3)=299**

|  |  |  |
| --- | --- | --- |
|  **X** |  **20** |  **3** |
|  **10** |  **200** |  **30** |
|  **3** |  **60** |  **9** |

 **230** Add the partial products (200 + 30) + (60 + 9) = 299

**+ 69**

 **299**

**Expanded long multiplication** (two-digit numbers multiplied by a teen-number):

**23 x 13 = 299**

 **23**

**X 13**

 **9 (3 x 3)**

 **60 (3 x 20)**

**+ 30 (10 x 3)**

 **200 (10 x 20)**

 **299**

This leads into….

**Compact long multiplication (formal method):**

**23 x 13 = 299**

 **23** Use the language of place value to ensure

**X 13** understanding.

**+ 69 ( 3 x 23)** Add the partial products.

 **230 ( 10 x 23)**

 **299**

Extend to larger two-digit numbers:

**56 x 27 = ( 50 + 6) x ( 20 + 7) = 1512**

|  |  |  |  |
| --- | --- | --- | --- |
|  **X** |  **50** |  **6** |  |
|  **20** |  **1000** |  **120** |  **1120** |
|  **7** |  **350** |  **42** |  **392** |

 **1512**

Add the partial products ( 1000 + 120) + ( 350 + 42) = 1512

**Expanded long multiplication** (two-digit numbers multiplied bt two-digit numbers):

**56 x 27 = 1512**

 **56**

**X 27**

 **42 (7 x 6)** This expanded method is linked to the

 **350 (7 x 50)** grid method.

**+ 120 (20 x 6)**

 **1000 (20 x 50)**

 **1512**

This leads into…

**Compact long multiplication (formal method):**

 **56 x 27 = 1512**

 **56** Use the language of place value to ensure

**X 27** understanding.

 **392 (7 x 56)** Add the partial products.

 **1120 (20 x 56)** The prompts (in brackets) can be omitted if

 **1512**  children no longer need them.

Extend with short and long multiplication of decimal numbers (initially in the context of money and measures), returning to an expanded method first, if necessary (see Y6 guidance).

**NB** If, at any time, children are making significant errors, return to the previous stage in calculation.

**Multiplication-Year Six**

* **Multiply multi-digit numbers (including decimals) up to four digits by a two-digit whole number**

**NB** Ensure that children are confident with the methods outlined in the previous year’s guidance before moving on.

Continue to practise and develop the **formal short multiplication** method and **formal long multiplication** method with larger numbers and decimals throughout Y6. Return to an expanded form of calculation initially, if necessary (see Y5 guidance).

**The grid method** (decimal number multiplied by a two-digit number):

**53.2 x 24 = 1276.8**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  **X** |  **50**  |  **3** |  **0.2** |  |
|  **20**  |  **1000** |  **60** |  **4** |  **1064.0** |
|  **4** |  **200** |  **12**  |  **0.8** |  **212.8** |
|  |  |  |  |  **1276.8** |

**The formal written method of long multiplication:**

 **53.2**

**X 24.0**  It is an option to include .0 in this example,

 **212.8 (53.2 x 4)** but not essential.

**1064.0 (53.2 x 20)**  The prompts (in brackets) can be omitted if

**1276.8** children no longer need them.

**NB** If, at any time, children are making significant errors, return to the previous stage in calculation.

Our aim is that, by the end of Y6, children use **mental methods (with jottings)** when appropriate, but for calculations nthaat they cannot do in their heads, they use an efficient **formal written method** accurately and with confidence.

 **STAGES IN DIVISION**

**Division-Early Stages (EYFS)**

Children will engage in a wide variety of songs and rhymes, games and activities. In practical activities and through discussion they will begin to solve problems involving halving and sharing.



Share the apples between two people.

“Half of the apples for you and half of the apples for me.”

**Division-Year One**

* **Solve one-step problems involving division by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher**
* **Count in multiples of twos, fives and tens** (to the tenth multiple).

Children will start with practical **sharing** using a variety of resources. They will share objects into **equal groups** in a variety of situations. They will begin to use the vocabulary associated with division in practical contexts.

“Share these eight apples equally between two children. How many apples will each child have?”

   

   

 

“Share twenty dogs into two groups. How many dogs are there in each group?”

Children will move from **sharing** to **grouping** in a practical way.

“Put twenty crayons into groups of ten. How many pots do we need?”

Use **arrays** to support early division.

    

    

“How many faces altogether? How many groups of two?”

    

    

 “Five groups of two.”

 “How many groups of 5?”

 “10 shared equally between two people.”

 “Half of ten is five.”



Continue to solve problems in **practical contexts** throughout Y1, and develop the language of early division, with appropriate resources.

**Division-Year Two**

* **Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables**
* **Calculate mathematical statements for division within the multiplication tables they know and write them using the division (÷) and equals (=) signs**
* **Solve problems involving division, using materials, arrays, repeated subtraction, mental methods, and multiplication and division facts, including problems in context.**

**NB** Ensure that children are confident with the methods outlined in the previous year’s guidance before moving on.

Children will use a range of vocabulary to describe division and use practical resources, pictures and diagrams and the **÷** sign to record, using multiples that they know.

**Sharing and grouping:**



“30 dogs shared equally into three groups.” (Sharing)

“We have thirty dogs and put ten dogs in each kennel. How many kennels do we need?” (Grouping)

“30 divided by 10 = 3”

“30 divided by 3 = 10”

**30 ÷ 10 = 3**

**30 ÷ 3 = 10**

“How many groups of five?”

“15 shared equally between 3 people is….?”

“15 divided by 3 equals 5.”

 “15 divided by 5 equals 3.”



**15 ÷ 5 = 3 15 ÷ 3 = 5**

**Use arrays to support division:**

**15 ÷ 5 = 3**

**15 ÷ 3 = 5**

 How many groups of three?

 **How many groups of five?**

15 shared between 3 people is ….?

 15 shared between 5 people is ….?

15 divided by 5 = 3

15 divided by 3 = 5

**When children are ready, use an empty number line to count forwards:**

**30 ÷ 5 = 6**

“How many jumps of five make thirty?”

 1 jump 2 jumps 3 jumps 4 jumps 5 jumps 6 jumps

 of 5 of 5 of 5 of 5 of 5 of 5

0 5 10 15 20 25 30

**Also jump back to make the link with repeated subtraction:**

**30 ÷ 5 = 6**

How many groups of 5?

 **-5 -5 -5 -5 -5 -5**

0 5 10 15 20 25 30

**NB** If, at any time, children are making significant errors, return to the previous stage in calculation.

**Division-Year Three**

* **Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables** (continue to practise the 2, 5 and 10 multiplication tables)
* **Write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers divided by one-digit numbers, using mental and progressing to a formal written method.**

**NB** Ensure that children are confident with the methods outlined in the previous year’s guidance before moving on.

Continue to use practical resources, pictures, diagrams, number lines, arrays and the **÷** sign to record, using multiples that they know, as appropriate.

**Using an empty number line to count forwards…**

**24 ÷ 3 = 8**

“How many threes in 24?”

 1 jump 2 jumps 3 jumps 4 lumps 5 jumps 6 jumps 7 jumps 8 jumps

 of 3 of 3 of 3 of 3 of 3 of 3 of 3 of 3

0 3 6 9 12 15 18 21 24

**….also jump back from 24 to make the link with repeated subtraction.**

 **-3 -3 -3 -3 -3 -3 -3 -3**

0 3 6 9 12 15 18 21 24

“How many groups of three in 24?”

**Introduce the formal layout using multiplication and division facts that the children know:**

**24 ÷ 3 = 8**

This can be recorded as…

 \_\_8\_\_\_

 3 | 24

“Twenty four divided by three equals eight.”

“How many threes are there in twenty four?”

**NB** If, at any time, children are making significant errors, return to the previous stage in calculation.

**Division-Year Four**

* **Recall multiplication and division facts for multiplication tables up to 12 x 12**
* **Use place value, known and derived facts to divide mentally**
* **Divide two-digit and three-digit numbers by a one-digit number using formal written layout (not explicitly stated in the programme of study but implied in the non-statutory guidance).**

**NB** Ensure that children are confident with the methods outlined in the previous year’s guidance before moving on.

Continue to write and calculate mathematical statements for division using the multiplication tables that the children know e.g

**32 ÷ 8 = 4**

Continue to use the **formal written layout** for division using multiplication tables that they know:

 \_\_\_\_4\_

 8 | 32

“How many eights are there in thirty two?”

Continue using the formal written layout, introducing remainders:

**25 ÷ 3 = 8 r1**

 **\_\_\_\_**8r1\_

3| 25

**NB** Remainders are not specifically referred to until Y5 in the National Curriculum. However, this may be an appropriate point to introduce them using familiar multiplication facts.

This could be modelled using an empty number line, if necessary:

**25 ÷ 3 = 8r1**

**0 1 4 7 10 13 16 19 22 25**

Alternatively you could jump forwards in multiples of three from zero to twenty four (“and one more makes 25”)

**Division using partitioning** (two digits divided by one digit):

**65 ÷ 5 = 13**

65 = 50 + 15 Partition 65 into 50 and 15

50 **÷** 5 = 10 15 **÷** 5 = 3 10 + 3 = 13

**NB** Children will need to practise partitioning in a variety of ways.

Continue to use **empty number lines**, as appropriate, using multiples of the divisor:

**65 ÷ 5 = 13**

-15 **(3**x 5) -50 (**10**x5)

**0 15 65**

**98 ÷ 7 = 14**

98 = 70 + 28 Partition 98 into 70 and 28

70 **÷** 7 = 10 28 **÷** 7 = 4 10 + 4 = 14

This could be modelled on an empty number line to further develop understanding.

**NB** Children will need to practise partitioning in a variety of ways.

**98 ÷ 7 = 14**

\_\_10 \_+\_\_4\_\_=\_\_14\_\_

 7 | 70 + 28

“We have partitioned 98 into 70 and 28 (90 = 70 + 28). Seven “goes into” 70 ten times and seven “goes into” 28 four times. Ten add four equals 14.”

**This will lead into the formal written method of short division:**

**98 ÷ 7 = 14**

\_\_\_\_\_14\_

 7 | 928 Use the vocabulary of place value to ensure

 understanding and make the link to

 partitioning.

Continue to practise the formal method of short division throughout Y4.

**If the children are confident**, develop further, by dividing three-digit numbers by a one-digit number using the formal method of short division with whole number answers (no remainders).

**NB** If, at any time, children are making significant errors, return to the previous stage in calculation.

**Division-Year Five**

* **Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.**

**NB** Ensure that children are confident with the methods outlined in the previous year’ guidance before moving on.

**Continue to practise the formal written method of short division with whole number answers…..**

**184 ÷ 8 = 23**

 \_\_\_23\_ Use the language of place value to ensure

 8 | 1824 understanding. Make the link to the partitioning

 Method (see Y4 guidance).

**…….and with remainders:**

**432 ÷ 5 = 86r2**

 \_\_\_86r2

 5 | 4332

The remainder can also be expressed as a fraction, 2/5 (the remainder divided by the divisor): **432 ÷ 5 = 86 2/5**

Continue to practise, develop and extend the formal method of short division, with and without remainders. Interpret and express remainders according to the context.

**NB** If, at any time, children are making significant errors, return to the previous stage in calculation.

**Division-Year Six**

* **Divide numbers up to 4 digits by a two-digit number using the formal method of short division where appropriate, interpreting remainders according to the context**
* **Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context.**

**NB** Ensure that the children are confident with the methods outlined in the previous year’s guidance before moving on.

Continue to practise the **formal method of short division,** with and without remainders, using the language of place value to ensure understanding (see Y5 guidance).

**496 ÷ 11 = 45r1**

 \_\_\_45r1

 11 | 4956

The remainder can also be expressed as a fraction, 1/11 (the remainder divided by the divisor).

Dividing by a two-digit number using a **formal method of long division:**

 \_\_\_\_\_45r1\_ Multiples of the divisor (11) have been

 11 | 496 subtracted from the dividend (496).

 - 440 (40 x 11) “40 (lots of 11) + 5 (lots of 11) = 45

 56 (lots of 11)”.

 - 55 (5 x 11) “1 is the remainder”.

 1 (remainder) Answer: 45 1/11

Standard short division does not help with the following calculation. However, it can be solved using **long division** (by repeated subtraction using multiples of the divisor):

**144 ÷ 16 = 9**

\_\_\_\_\_\_\_9

 16 | 144 Multiples of the divisor (16) have been

 - 64 (4x 16) subtracted from the dividend (144).

 80 “4 (lots of 16) + 4 (lots of 16) + 1 (lot of 16)

 - 64 (4 x 16) = 9 (lots of 16).

 16 There is no remainder.”

 - 16 (1 x16)

 0

Children will need to select the most effective method for each calculation/problem they meet, including whether to use the standard, **formal written method of long division:**

**432 ÷ 15 = 28r12**

 \_\_\_\_28r12 Multiple of the divisor (15) have been

 15 | 432 subtracted from the dividend (432)

 - 300 (20 x 15) “20 (lots of 15) + 8 (lots of 15) = 28.

 132 12 is the remainder.”

 - 120 ( 8 x 15)

 12 (remainder)

The remainder can also be expressed as a fraction, 12/15 (the remainder divided by the divisor) or as a decimal. **0.8** (see next example).

The answer is: 28 12/15 or 28.8

This is an alternative way to record formal long division:

**432 ÷ 15 = 28.8**

 \_\_\_\_28.8\_ **NB** Only teach this method when children are

 15 | 432.0 completely secure with the previous method.

 30 The remainder is expressed as a decimal.

 132

 120

 120

 120

 0

**NB** If, at any time, children are making significant errors, return to the previous stage in calculation.

Our aim is that, by the end of Y6, children use **mental methods (with jottings)** when appropriate, but for calculations that they cannot do in their heads, they use an efficient **formal written method** accurately and with confidence.